Hall Ticket Number:

II/IV B.Tech (REGULAR/SUPPLEMENTARY) DEGREE EXAMINATION

April ,2017 **Electronics and Instrumentation Engineering**

Fourth Semester **Signals & Systems**

Time: Three Hours **Maximum :** 60 Marks

Answer question number 1 compulsory (1 x 12 = 12 Marks)

Answer one question from each unit. (4 x 12 = 48 Marks)

1. Answer all questions. (1 x 12 = 12 Marks)
   a. Define symmetric and anti symmetric signals?
   b. When a signal is said to be causal signal?
   c. Compare energy and power signals?
   d. Define LTI system?
   e. Give the relation between impulse response and step response of a system?
   f. What are the necessary conditions for the stability of a system?
   g. Define white noise?
   h. State the Parsval's theorem?
   i. Define noise figure?
   j. Define conditional probability?
   k. Write the properties of Gaussian density function?
   l. Write any two properties of probability density function?

2 **a.** Derive the relationship between trigonometric Fourier series,+ and exponenetial Fourier series. 6M

b. A rectangular function defined by $f(t) = \begin{cases} 1 & 0 \le t < \pi \\ -1 & \pi \le t < 2\pi \end{cases}$ 6M

Approximate a function by single sinusoid "sin(t) ". Evaluate mean square value of the error in this approximation. Also show what happens when more sinusoidal terms are used for approximation.

(OR)

3 **a.** Find fourier trnasform of impulse, Gate fnction of width 'T' and amplitude 1. 6M

b. State and prove sampling theorem. 6M

4. **a.** Derive the expression for convolution sum. 6M

b. Find the convolution of the following signals x[n] = {1,2,3,4} and h[n]={3,4,5}. 6M

(OR)

5. **a.** Find the frequency response of an LTI system described by the differential equation 6M

$$\frac{d^2y(t)}{dt^2} + 5\frac{dy(t)}{dt} + 6y(t) = 2x(t)$$

b. Write the properties of autocorrelation and cross correlation funcction. 6M

6. **a.** Prove the available thermal noise power density from any passive RLC two terminal network is a constant. 6M

b. Explain how to express noise figure in terms of available power densities and available power gain, 6M

(OR)

7. **a.** Derive the expression for noise figure of a multistage amplifier. 6M

b. Derive the expression for RMS value of noise voltage in RC circuit. 6M

8. a. With an example illustrate Bayes theorem and conditional probability. 6M

b. A missile can be acccendentally launcehed if two relays A and B both have failed. The probability of A and B failing are known to be 0.01 and 0.03 respectively. It is also known that B is more likely to fasil (probability 0.06) if A has failed. 6M

i. What is the probability of an accendental missile launch.

ii. What is the probability that A will fail if B has failed.

iii. Are the events A fails and B fails statistically independent.

(OR)

9. a. What is the probabiliy density function of a random variable? Write the properties of probability dwnsity function? 6M

b. A random variable X has a probability density 6M

$$f_X(x) = \begin{cases} \frac{1}{2}\cos(x) & -\frac{\pi}{2} < x < \frac{\pi}{2} \\ 0 & else \quad where \quad in \quad x \end{cases}$$

derive

Find the mean value of $g(x) = 4X^2$

II/IV B.Tech (REGULAR/SUPPLEMENTARY) DEGREE EXAMINATION

April ,2017 **Electronics and Instrumentation Engineering**

Fourth Semester **Signals & Systems**

SCHEME OF EVALUATION

1, a. Define symmetric and anti symmetric signals? 1M

Ans. A g(t) which follows the property that g(t) = g(-t) is known as symmetric signal.
A g(t) which follows the property that g(t) = -g(-t) is known as Anti-symmetric signal.

b. When a signal is said to be causal signal? 1M

Ans. A signal g(t) is said to be causal if g(t) = 0 for t<0 similarly for discrete signals g[n] = 0 for n<0;

c. Compare energy and power signals? 1M

Ans. A signal is said to be energy signal if and only if its total energy is finite i.e. $(0<E<\infty)$.
Average power of the energy signal is 0; Non-periodic signals are energy signals.
A signal is said to be power signal if and only if its average power is finite i.e. $(0<P<\infty)$.
The energy of a power signal is ∞; Periodic signals are energy signals.

d. Define LTI system? 1M

Ans. A system which follows the principle of superposition and homogeneity and in addition the input and output characteristics do not change with time is called LTI system.

e. Give the relation between impulse response and step response? 1M

Ans. Unit step response of a signal *s(t)* is
*s(t) = h(t) * u(t) = u(t) *h(t);*

$$s(t) = \int_{-\infty}^{t} h(\tau)d\tau$$

It shows that unit step response of a continuous time system is the running integral of its impulse response.

f. What are the necessary conditions for the stability of a system? 1M

Ans. A system is said to be stable for a bounded input produces bounded output and its impulse response h(t) must be absolutely integral i.e. $\int_{-\infty}^{+\infty} |h(\tau)| d\tau < \infty$

g. Define white noise? 1M

Ans. White Noise is the noise that has constant magnitude of power over frequency.
Examples of White Noise are Thermal Noise, and Shot Noise.

h. State the Parsval's theorem? 1M

Ans. Parsval's theorem states that the energy of a signal g(t) can be evaluated directly from the frequency spectrum G(ω) without the knowledge of time domain signal.

$$E = \int_{-\infty}^{\infty} |g(t)|^2 dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |G(\omega)|^2 d\omega$$

$$E = \int_{-\infty}^{\infty} |G(f)|^2 df$$

i. Define noise figure? 1M

Ans. **Noise figure** (NF) and **noise factor** (F) are measures of degradation of the signal-to-**noise** ratio (SNR), caused by components in a radio-frequency (RF) signal chain. It is a number by which the performance of an amplifier or a radio receiver can be specified, with lower values indicating better performance.

j. **Define conditional probability?** 1M

Ans. The conditional probability of an event A assuming another event B, denoted by P(A|B),

$P(A \mid B) = \frac{P(AB)}{P(B)}$ where we assume that P(B) ≠0;

k. **Write the properties of Gaussian density function?** 1M

Ans. Gaussian Density function:

$$f_X(x) = \frac{1}{\sqrt{2\pi\sigma}} \exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right], \qquad -\infty < x < \infty$$

The function has bell shaped curve
It is distributed evenly over the mean 'μ';
The variance of the function σ2.
The function is completely characterized by first and second moments.

l. **Write any two properties of probability density function?** 1M

Ans.
1. The derivative of the probability distribution function $F_X(x)$ is called probability density function. $f_X(x) = \frac{dF_X(x)}{dx}$.

Any two properties only

2. $\int_{-\infty}^{+\infty} f_X(x)dx = 1$
3. $P\{x_1 < x \le x_2\} = F_X(x_2) - F(x_1) = \int_{x_1}^{x_2} f_X(x)dx$
4. $f_X(x) = \sum_i p_i \delta(x - x_i)$

2 **a.** **Derive the relationship between trigonometric Fourier series, and exponenetial Fourier series.**

Ans. Fourier series: Any periodic signal f(t) of period Tis represented as the sum of din and cosine harmonics as given by

$$f(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t)$$

Where $a_0 = \frac{1}{T}\int_T f(t)dt$ **2M**

$$a_n = \frac{2}{T}\int_T f(t)\cos(n\omega_0 t)dt$$

$$b_n = \frac{2}{T}\int_T f(t)\sin(n\omega_0 t)dt$$

Exponential Fourier Series:

$$f(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t}$$

$$C_n = \frac{1}{T}\int_T f(t)e^{-jn\omega_0 t}$$

After rearranging both the seriates then

$$f(t) = a_0 + a_1\cos(n\omega_0 t) + a_2\cos(n\omega_0 t) + ... + a_n\cos(n\omega_0 t)$$

$$............. + b_1\sin(n\omega_0 t) + b_2\sin(n\omega_0 t) + ... + b_n\sin(n\omega_0 t)$$ **2M**

For the exponential series:

$$f(t) = ... + C_{-r}e^{(-jr\omega_0 t)} + ... + C_{-2}e(-j2\omega_0 t) + C_{-1}e^{(-j\omega_0 t)} + C_0 +$$

$$C_1 e^{(j\omega_0 t)} + C_2 e^{(j2\omega_0 t)} + ... + C_r e^{(jr\omega_0 t)} + ...$$

Substituting the relation in trigonometric F.S.

$$\cos(n\omega_0 t) = \frac{e^{jn\omega_0 t)} + e^{-jn\omega_0 t)}}{2}$$ **2M**

$$\sin(n\omega_0 t) = \frac{e^{jn\omega_0 t)} - e^{-jn\omega_0 t)}}{2j}$$

Then we will get

$$C_0 = a_0$$

$$C_n = \frac{1}{2}(a_n - jb_n)$$

$$C_{-n} = \frac{1}{2}(a_n + jb_n)$$

b. **A rectangular function defined by** $f(t) = \begin{cases} 1 & 0 \le t < \pi \\ -1 & \pi \le t < 2\pi \end{cases}$

Approximate a function by single sinusoid "sin(t) ". Evaluate mean square value of the error in this approximation. Also show what happens when more sinusoidal terms are used for approximation.

Ans. $f(t) = C_1\sin(t)$ **3M**

$$C_1 = \frac{\int_0^{2\pi} f(t)\sin(t)dt}{\int_0^{2\pi}\sin^2(t)dt}$$

Denominator= $\int_0^{2\pi}\sin^2(t)dt = \int_0^{2\pi}\frac{1-\cos(2t)}{2}dt = \frac{1}{2}\int_0^{2\pi}[1-\cos(2t)]dt$

$$= \frac{1}{2}\int_0^{2\pi}[1-\cos(2t)]dt = \frac{1}{2}\left[t - \frac{\sin(2t)}{2}\right]_0^{2\pi} = \frac{1}{2}(2\pi) = \pi$$

Numerator =

$$\int_0^{2\pi} f(t)\sin(t)dt = \int_0^{\pi}\sin(t)dt \text{ - } \int_{\pi}^{2\pi}\sin(t)dt$$

$$\left[-\cos(t)\right]_0^{\pi} - \left[-\cos(t)\right]_{\pi}^{2\pi} => C_1 = \frac{4}{\pi}$$

$$f(t) = \frac{4}{\pi}\sin(t)$$

Similarly if we consider third and fifth harmonic then

$$C_3 = \frac{4}{3\pi}$$

$$C_5 = \frac{4}{5\pi}$$

Then the mean square value of error if we consider different harmonics then the error in representation will reduced and ripples will present in peaks. **3M**

$$\left(\int_T f^2(t)dt\right)$$

$$T = 2\pi,$$

$$= \left(\int_0^{\pi} dt + \int_{\pi}^{2\pi} dt\right) = (2\pi)$$

$$= \frac{1}{2\pi}\left(2\pi - \left(\frac{4}{\pi}\right)^2 \pi\right)$$

$$= 0.189$$

$$MSE = \frac{1}{T}\left(\int_T f^2(t)dt - \left[C_1^2 K_1 + C_2^2 K_2 + \ldots\ldots + C_n^2 K_n\right]\right)$$

$$MSE_1 = \frac{1}{T}\left(\int_T f^2(t)dt - \left[C_1^2 K_1\right]\right)$$

$$MSE_2 = 0.099$$

$$MSE_3 = 0.066$$

(OR)

3 a. Find fourier trnasform of impulse, Gate fnction of width 'T' and amplitude 1.

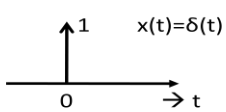
Ans. Fourier transform of impulse function: $\delta(t)$

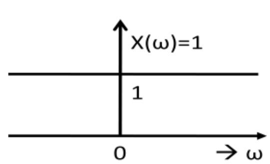
3M

$$\delta(t) = \begin{cases} 1 & for \quad t = 0 \\ 0 & for \quad t \neq 0 \end{cases}$$

$$X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt = \int_{-\infty}^{\infty} \delta(t)e^{-j\omega t}dt$$

$$= e^{-j\omega t}\Big|_{t=0} = 1$$



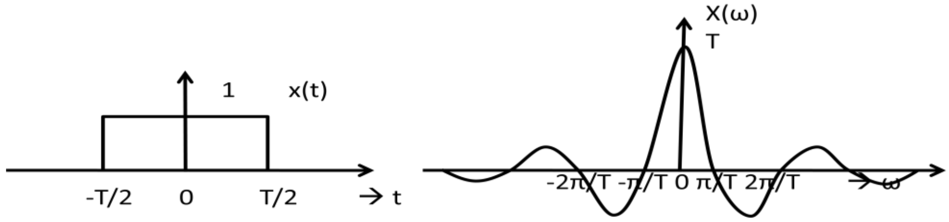


Fourier trnasform of gate function of width T and amplitude 1.

$$x(t)=\begin{cases}1 & for \quad \frac{-T}{2}<t<\frac{T}{2}\\ 0 & for \quad |t|>\frac{T}{2}\end{cases}$$

$$X(\omega)=\int_{-T/2}^{T/2}x(t)e^{-j\omega t}dt=\int_{-T/2}^{T/2}e^{-j\omega t}dt=TSa\left(\frac{\omega T}{2}\right)$$

3M



b. State and prove sampling theorem.

Ans.
1) A band limited signal of finite energy , which has no frequency components higher than W hertz , is completely described by specifying the values of the signal at instants of time separated by (1/2W) seconds **1M**
2) A band limited signal of finite energy, which has no frequency components higher than W hertz , may be completely recovered from the knowledge of its samples taken at the rate of 2W samples per second.

Proof of sampling theorem

There are two parts :

i. Representation of x(t) in terms of its samples **3M**

ii. Reconstruction of x(t) from its samples

PART I: Representation of x(t) in its samples x(nTs) x(nTs)
Step 1 : Define $x_\delta(t)$
Step 2 : Fourier transform of $x_\delta(t)$ has $X_\delta(f)$
Step 3: Relation between X(f) and $X_\delta(f)$
Step 4 : Relation between x(t) and $x(nT_s)$

Step 1 : Define $x_\delta(t)$
The sampled signal $x_\delta(t)$ is given as ,

$$x_\delta(t)=\sum_{n=-\infty}^{\infty}x(t)\delta(t-nT_s)$$

$$=\sum_{n=-\infty}^{\infty}x(nT_s))\delta(t-nT_s) \qquad ......(1)$$

$x(nT_s)$ is basically $x(t)$ sampled at t= nT_s , n=0,$\pm1,\pm2,\pm3,...$

Step 2 : Fourier transform ofx_δ (t)i.e. $X\delta(f)$

$$X_\delta(f)=FT\{\ \sum_{n=-\infty}^{\infty}x(t)\delta(t-nT_s)\ \} \qquad ....(2)$$

=FT {Product of $x(t)$ and impulse train}

We know that FT of product in time domain becomes convolution in frequency domain i.e.,

$X_\delta(f) = FT\{x(t)\} * FT\{\delta(t - nT_s)\}$

By definitions $x(t) \leftrightarrow X(f)$ and

$$\delta(t - nT_s) \leftrightarrow f_s \sum_{n=-\infty}^{\infty} \delta(f - nf_s)$$

Hence equation (2) becomes,

$$X_\delta(f) = X(f) * f_s \sum_{n=-\infty}^{\infty} \delta(f - nf_s)$$

Since convolution is linear,

$$X_\delta(f) = f_s \sum_{n=-\infty}^{\infty} X(f) * \delta(f - nfs)$$

$$= f_s \sum_{n=-\infty}^{\infty} X(f - nf_s) \ldots\ldots \text{By shifting property of impulse function}$$

$$= \ldots f_s X(f - 2f_s) + f_s X(f - f_s) + f_s X(f) + f_s X(f - f_s) + f_s X(f - 2f_s) + \ldots$$

Important assumption

Let us assume that fs=2W ,then

$X_\delta(f) = f_s X(f) \text{ for } -W <= f <= W \text{ and } f_s = 2W$ ......(3)

Or

$$X(f) = \frac{1}{f_s} X_\delta(f)$$

Step 4 : Relation between $x(t)$ and $x(nT_s)$

DTFT is $X(\Omega) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\Omega n}$

$\therefore X(f) = \sum_{n=-\infty}^{\infty} x(n) e^{-j2\pi fn}$ ....(4)

In above equation $'f'$ is the frequency of DT signal .If we replace $X(f)$ by $X_\delta(f)$, then $'f'$ becomes frequency of CT signal .i.e.,

$X_\delta(f) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j2\pi \frac{f}{f_s} n}$

$$\therefore X(f) = \frac{1}{f_s} \sum_{n=-\infty}^{\infty} x(n) e^{-j2\pi f n T_s}$$

Inverse Fourier Transform (IFT) of above equation gives $x(t)$ i.e.,

$x(t) = IFT\left\{\frac{1}{f_s} \sum_{n=-\infty}^{\infty} x(n) e^{-j2\pi f n T_s}\right\}$.........(5)

Conclusions:

1) Here x(t) is represented completely in terms of $x(nT_s)$

2) Above equation holds for fs=2W.This means if the samples are taken at the rate of 2W or higher, x(t) is completely represented by its samples. **2M**

ii) Reconstruction of x(t)from its samples

Step 1 : Take inverse Fourier transform of X(f) which is in terms of X_δ (f)

Step 2 : Show that x(t) is obtained back with the help of interpolation function.

Take inverse Fourier transform of the above

$$x(t) = \int_{-\infty}^{\infty} \{\frac{1}{f_s} \sum_{n=-\infty}^{\infty} x(n) e^{-j2\pi f n T_s}\} e^{j2\pi f t} df$$

Here the integration can be taken from -W$\leq f \leq W$. Since $\therefore X(f) = \frac{1}{f_s} X_\delta(f)$ for W$\leq f \leq W$

Refer fig.4.1.2.

$$\therefore x(t) = \int_{-W}^{W} \frac{1}{f_s} \sum_{n=-\infty}^{\infty} x(n) e^{-j2\pi f n T_s} e^{j2\pi f t} df$$

Interchanging the order of summation and integration,

$$x(t) = \sum_{n=-\infty}^{\infty} x(nT_s) \frac{1}{f_s} \int_{-W}^{W} e^{j2\pi f(t-n\, T_s)} df$$

$$= \sum_{n=-\infty}^{\infty} x(nT_s) \frac{1}{f_s} \left[\frac{e^{j2\pi f(t-nT_s)}}{j2\pi((t-nT_s)}\right] from - WTOW$$

$$= \sum_{n=-\infty}^{\infty} x(nT_s) \frac{1}{f_s} \{\frac{e^{j2\pi W(t-nT_s)} - e^{-j2\pi W(t-nT_s)}}{j2\pi(t-nT_s)}\}$$

$$= \sum_{n=-\infty}^{\infty} x(nT_s) \frac{1}{f_s} \frac{sin 2\pi W(t-nT_s)}{\pi(t-nT_s)}$$

$$= \sum_{n=-\infty}^{\infty} x(nT_s) \frac{\sin\pi(2Wt-2WnT_s)}{\pi(f_s t - f_s n T_s)}$$

Here f_s =2W, hence$T_s = \frac{1}{f_s} = \frac{1}{2W}$

Simplifying above equation,

Conclusions:

The samples x(nTs)are weighted by sinc functions.
The sinc function is the interpolating function

$$x(t) = \sum_{n=-\infty}^{\infty} x(nT_s) \frac{\sin\pi(2Wt-n)}{\pi(2Wt-n)}$$

$$= \sum_{n=-\infty}^{\infty} x(nT_s)\, sinc(2Wt-n)$$

Since $\frac{\sin\pi\theta}{\pi\theta} = sinc\theta$ ....(6)

Step 2:

Let us interpret the above equation .Expanding we get ,

$$x(t) = ..+x(-2T_s)sinc(2Wt+2) + x(-T_s)sinc(2Wt+1) + x(0)sinc(2l + x(T_s)sinc(2Wt-1) + \cdots$$

Conclusions:

The samples x(nTs(nTs)are weighted by sinc functions.
The sinc function is the interpolating function

4. a. Derive the expression for convolution sum.

Ans. Consider LTI system **6M**

Initially relaxed at t=0;

If the input of the system is impulse

Then the out put of the system is denoted by h(t) and is called impulse response.
h(t) =T[δ(t)];

$$x(t) = \int_{-\infty}^{\infty} x(\tau)\delta(t-\tau)d\tau$$

$$y(t) = T[x(t)]$$

$$y(t) = T\left[\int_{-\infty}^{\infty} x(\tau)\delta(t-\tau)d\tau\right]$$

$$y(t) = \left[\int_{-\infty}^{\infty} x(\tau)T[\delta(t-\tau)]d\tau\right]$$

$$h(t,\tau) = T[\delta(t-\tau)]$$

$$y(t) = \left[\int_{-\infty}^{\infty} x(\tau)h(t-\tau)]d\tau\right]$$

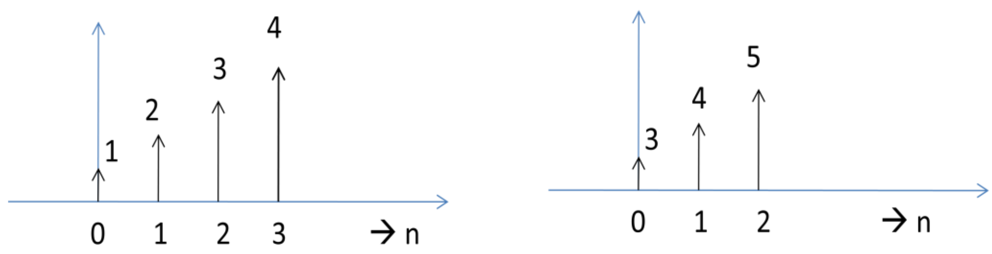
This is known as convolution summation. The convolution of any two signals is given by
*y(t) =x(t)*h(t)*

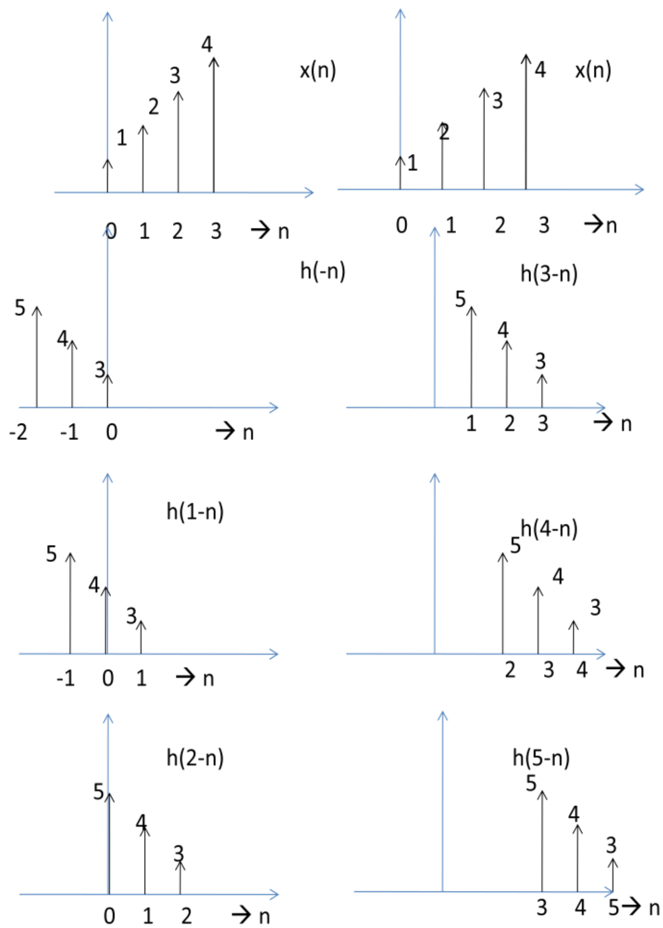
similarly we can prove for the discrete systems

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

b. Find the convolution of the following signals x[n] = {1,2,3,4} and h[n]={3,4,5}.

Ans. x([n] = { 1,2,3,4} h[n] = { 3,4,5} **6M**





$$y(n) = \sum_{k=-2}^{3} x(k)h(n-k)$$

$$y(0) = \sum_{k=-2}^{3} x(k)h(-k) = 3$$

$$y(1) = \sum_{k=-2}^{3} x(k)h(1-k) = 10$$

$$y(2) = \sum_{k=-2}^{3} x(k)h(2-k) = 22$$

$$y(3) = \sum_{k=-2}^{3} x(k)h(3-k) = 34$$

$$y(4) = \sum_{k=-2}^{3} x(k)h(4-k) = 31$$

$$y(5) = \sum_{k=-2}^{3} x(k)h(5-k) = 20$$

Y[n]={3,10,22,34,31,20}

(OR)

5. a. Find the frequency response of an LTI system described by the differential equation $\frac{d^2y(t)}{dt^2} + 5\frac{dy(t)}{dt} + 6y(t) = 2x(t)$

Ans. Find the laplace transform of the above differential equation 6M

$$s^2Y(s) + 3sY(s) + 6Y(s) = 2X(s)$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{2}{s^2+5s+6} = 2\left(\frac{1}{(s+2)} - \frac{1}{(s+3)}\right)$$

Impulse ressponse is given by

$h(t) = 2\left(e^{-2t} - e^{-3t}\right)$

b. Write the properties of autocorrelation and cross correlation funcction.

Ans. Cross correlation : 3M

1. The cross correlation function exhibits cojugate symmetry

$$R_{12}(\tau) = R^*_{21}(-\tau)$$

2. If $R_{12}(0) = 0$ then $\int_{-\infty}^{\infty} x_1(t)x_2^*(t)dt = 0$ then the two signals are said to be orthogonal over the entire time interval.
3. The cross correlation of two energy signals is the multiplication of the fourier trnasform of one signal with complex cojugate of the Fourier transform of the second signal.

$$R_{12}(\tau) \leftrightarrow X_1(\omega)X_2^*(\omega)$$

Auto correlation:

Autocorrelation of energy signal x(t) is given by

$$R_{11}(\tau)=R(\tau)= \int_{-\infty}^{\infty} x(t)x^*(t-\tau)dt = 0$$ 3M

1. The cross correlation function exhibits cojugate symmetry

$$R(\tau) = R^*(-\tau)$$

2. The value of auto correlation function at the orign is equal to the total energy of the signal.

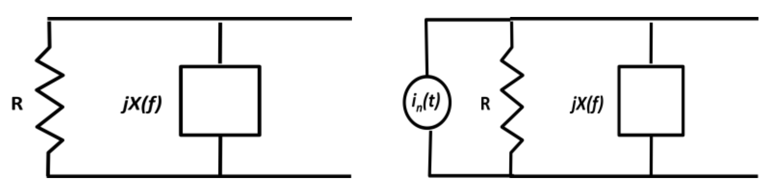
$$R(0) = \int_{-\infty}^{\infty} x(t)x^*(t)dt = \int_{-\infty}^{\infty} |x(t)|^2 dt = E$$

3. Autocorrelation function has the maximum value at the orign.
4. Autocorrelation function and energy spectral density funtion forms fourier transform pairs.

$$R(\tau) \leftrightarrow \psi(\omega)$$

6. a. Prove the available thermal noise power density from any passive RLC two terminal network is a constant.

Ans. Consider the following passive circuit **6M**

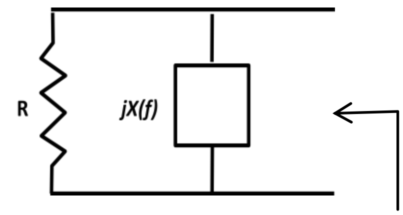


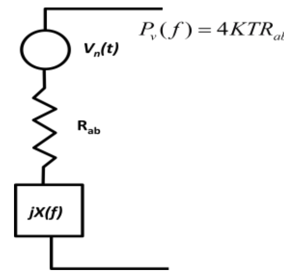
$$P_{th}(f) = 4KTG = 4KT/R$$

$$H(j\omega) = \frac{jRX(f)}{R + jx(f)} \qquad |H(j2\pi f)|^2 = \frac{R^2X^2(f)}{R^2 + X^2(f)}$$

$$P_o(f) = \left(\frac{4kT}{R}\frac{R^2X^2(f)}{R^2+X^2(f)}\right) = \left(4kT\frac{RX^2(f)}{R^2+X^2(f)}\right)$$

The Thevinin's equivalent of the above circuit is





Impedance of the network is given by

$$Z_{ab}(f) = \frac{jRX(f)}{R + jX(f)} = \frac{RX^2(f)}{R^2 + X^2(f)} + j\frac{R^2 X(f)}{R^2 + X^2(f)} = R_{ab}(f) + jX_{ab}(f)$$

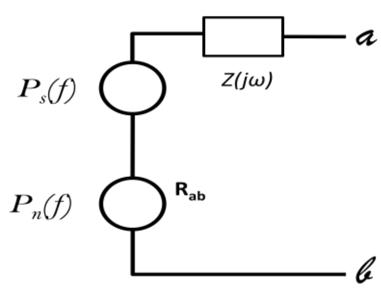
$$P_0(f) = 4KT\frac{RX^2(f)}{R^2 + X^2(f)}$$

$$P_0(f) = 4KTR_{ab}(f)$$

b. **Explain how to express noise figure in terms of available power densities and available power gain,**

Ans. Noise figure is expressed as the rario of the signal to noise power density spectra at the input divided by that at the output. **6M**

Cosider the following circuit as shown below.



$$\boxed{(S/N)_i = \frac{P_s(f)}{P_n(f)}}$$

$$\frac{P_s(f)}{[2R(f)]^2}R(f) = \frac{P_s(f)}{[4R(f)]}$$

Available noise power density is

$$\frac{P_n(f)}{[2R(f)]^2}R(f) = \frac{Pn(f)}{[4R(f)]}$$

$$\left(\frac{S}{N}\right)_{av} = \frac{P_s(f)}{P_n(f)}$$

$$F = \frac{(P_{si})_{av} / (P_{nsi})_{av}}{(P_{so})_{av} / (P_{nto})_{av}}$$

$$g = \frac{available \quad signal \quad power \quad density \quad at \quad the \quad output}{available \quad signal \quad power \quad density \quad at \quad the \quad input} = \frac{(P_{so})_{av}}{(P_{si})_{av}}$$

$$F = \frac{(P_{nto})_{av}}{g(P_{nsi})_{av}},$$

$(P_{nsi})_{av}$=KTn

$$F = \frac{(P_{nto})_{av}}{gKT_n}$$

$$(P_{nto})_{av} = (P_{nso})_{av} + (P_{nao})_{av}$$

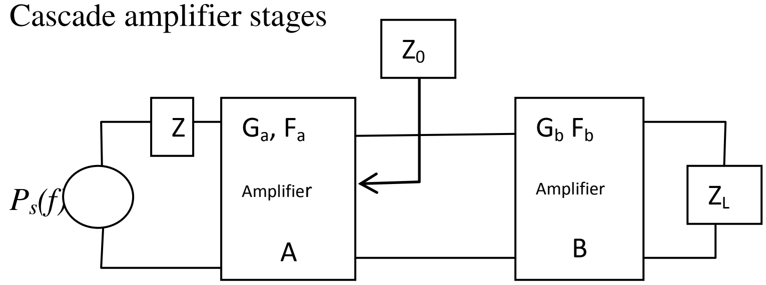
By substituting (Pnto)av

We will get the following expression $(P_{nao})_{av} = (F\text{-}1)gkT$

(OR)

7. a. Derive the expression for noise figure of a multistage amplifier.

Ans. Cascade amplifier stages **3M**



$(P_{nao})_{av} = (F\text{-}1)gkT$

$(P_{nto})_{av}=F_{ab}G_{ab}kT$ **2M**

$(P_{nto})_{av}=F_{ab}G_aG_bkT$

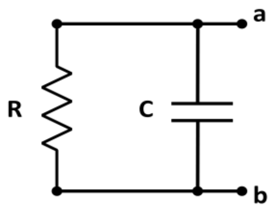
$(P_{nto})_{av}=F_{ab}G_aG_bkT= (F_b\text{-}1)G_bkT+ F_aG_aG_bkT$

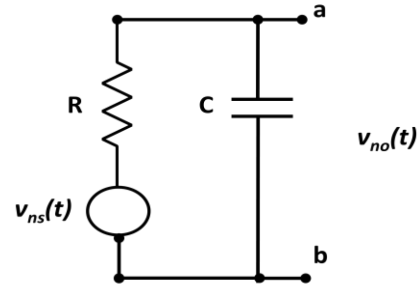
$F_{ab} = F_a + \frac{F_b - 1}{G_a}$ *and in general*

$$F = F_a + \frac{F_b - 1}{G_a} + \frac{F_c - 1}{G_a G_b} + ...$$ **1M**

b. Derive the expression for RMS value of noise voltage in RC circuit.

Ans. **2M**





$$H(\omega) = \frac{1}{j\omega RC + 1}. \qquad H(f) = \frac{1}{j2\pi fRC + 1}.$$

The power density at the output terminals is $P_o(f)$ given by **3M**

$$P_o(f) = P_i(f)\left|H(j\pi f)\right|.^2$$

$$= \frac{4kTR}{1 + 4\pi^2 f^2 R^2 C^2}.$$

Root mean square value of the voltage is given by

$$\sqrt{\overline{v}_{no}^2} = \left[\int_0^\infty \frac{4kTR}{1 + 4\pi^2 f^2 R^2 C^2}\, df\right]^{1/2}$$

$$= \left[\frac{4kTR}{2\pi RC} \tan^{-1}(2\pi fRC)\Big|_0^\infty\right]^{1/2} = \sqrt{\frac{kT}{C}}$$ **1M**

Thus the RMS value of the noise voltage of passive circuit is constant.

8. a. With an example illustrate Bayes theorem and conditional probability.

Ans. Bayes' Theorem: **4M**

Let B1, B2, ·, Bk be a collection of k mutually exclusive and exhaustive events. Then for any event A with P(A) > 0 we have

$$P(B_n / A) = \frac{P(B_n \cap A)}{P(A)}$$

If P(A) ≠ 0 then

$$P(A / B_n) = \frac{P(A \cap B_n)}{P(B_n)}$$

$P(B_n) \neq 0$; by equating the above two expressions we can write

$$P(B_n / A) = \frac{P(A | B_n) P(B_n)}{P(A)}$$

By using the toal probabilities we can also be expressed as

$$P(B_n / A) = \frac{P(A | B_n) P(B_n)}{P(A / B_1) P(B_1) + P(A / B_2) P(B_2) + \ldots\ldots + P(A / B_n) P(B_n)}$$

Eg:

An urn B1 contains 2 white and 3 black balls and another urn B2 contains 3 white and 4 black balls. One urn is selected at random and a ball is drawn from it. If the ball drawn is found black, find the probability that the urn chosen was B1. **2M**

Let E_1 and E_2 are the events of selecting urns B_1 and B_2 respectively.

$P(E_1)=\frac{1}{2}, P(E_2)=\frac{1}{2}$

Let B denote the event that the ball choosen form the urn is black.
Then we can find event $P(E_1|B)$
$P(B|E_1)=3/5$ and $P(B|E_2)=4/7$
By using baye's theporem

$$P(E_1 \mid B)=\frac{\mathrm{P(E_1)P(B/E_1)}}{\mathrm{P(E_1)P(B/E_1)+P(E_2)P(B/E_2)}}=\frac{(1/2)(3/5)}{(1/2)(3/5)+(1/2)(4/7)}=\frac{3/10}{3/10+4/14}=\frac{21}{41}$$

b. A missile can be acccendentally launcehed if two relays A and B both have failed. The probability of A and B failing are known to be 0.01 and 0.03 respectively. It is also known that B is more likely to fasil (probability 0.06) if A has failed.

i. What is the probability of an accendental missile launch.
ii. What is the probability that A will fail if B has failed.
iii. Are the events A fails and B fails statistically independent.

Ans. Probabiility of failure of A =0.01. **6M**
Probabiility of failure of B =0.03.
Probability of failure of B given the failure of A = 0.06

1. probability of failue ar launch of missile accendentally took place is
P(A).P(B|A)+P(B).P(A|B) = 0.01*0.06+0.03*0.02=0.0006+0.0006=0.0012.

2. Probability od A will fail if B will fail

$$P(B/A)=\frac{P(A/B)P(B)}{P(A)}$$

$$P(A/B)=\frac{P(B/A)P(A)}{P(B)}=\frac{0.06\times 0.01}{0.03}=0.02$$

3. the A fials and B fails are not statisitcally independent.

(OR)

9. a. What is the probabiliy density function of a random variable? Write the properties of probability dwnsity function?

Ans. **6M**

1. The derivative of the probability distribution function $F_X(x)$ is called probability density function. $f_X(x)=\frac{dF_X(x)}{dx}$.
2. $\int_{-\infty}^{+\infty} f_X(x)dx=1$
3. $P\{x_1 < x \le x_2\}=F_X(x_2)-F(x_1)=\int_{x_1}^{x_2} f_X(x)dx$
4. $f_X(x)=\sum_i p_i\delta(x-x_i)$

b. **A random variable X has a probability density**

$$f_X(x) = \begin{cases} \frac{1}{2}\cos(x) & -\frac{\pi}{2} < x < \frac{\pi}{2} \\ 0 & else \quad where \quad in \quad x \end{cases}$$

Find the mean value of *g(x) = 4X²*

Ans. The mean value of g(x) is diven by E[*g(x)*] **6M**

$$E[g(x)] = E[4X^2] = \int_{-\infty}^{\infty} 4x^2 f_X(x)dx$$

$$= \int_{-\pi/2}^{\pi/2} 4x^2 \frac{1}{2}\cos(x)dx$$

$$= 2\left[x^2 \int \cos(x)dx - \int 2x \int \cos(x)dxdx\right]_{-\pi/2}^{\pi/2}$$

$$= 2\left[x^2 \sin(x) - \int 2x\sin(x)dx\right]_{-\pi/2}^{\pi/2}$$

$$= 2\left[x^2 \sin(x) - 2\left[x\int \sin(x)dx - \int\int \sin(x)dxdx\right]\right]_{-\pi/2}^{\pi/2}$$

$$= 2\left[x^2 \sin(x) - 2\left[x(-\cos(x) - \int(-\cos(x)dx\right]\right]_{-\pi/2}^{\pi/2}$$

$$= 2\left[x^2 \sin(x) - 2\left[x(-\cos(x) + \sin(x)\right]\right]_{-\pi/2}^{\pi/2}$$

$$= 2\left[\left(\frac{\pi}{2}\right)^2 (2) - 4\right] = \pi^2 - 8$$

II/IV B.Tech (REGULAR/SUPPLEMENTARY) DEGREE EXAMINATION

SCHEME OF EVALUATION

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Fourth Semester
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Subject Name: Signals & Systems

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